FailRecOnt -- An Ontology-based Approach for Failure Interpretation and Recovery in Planning and Execution

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Description Logic Primer

We will give a short introduction to Description Logic, and specifically its "ALCHI" fragment, here; for more details see [15].

Description Logic (DL) is a first order logic formalism used in various knowledge representation and reasoning applications because it provides a good balance of expressiveness and tractability. Many of its fragments are decidable, and well-optimized reasoners exist.

DL makes a distinction between *concepts*, also sometimes known as *classes*, and *individuals*; an entity cannot be both. From this distinction, another follows: an axiom is either terminological or an individual assertion.

Terminological axioms define concepts and subsumption relations between them. Together, terminological axioms form the *T-box* of an ontology. Individual *assertions* state to which concept(s) and individual belongs to, whether two individuals are the same or different, and what relations exist between individuals. Together, assertions form the *A-box* of an ontology.

Relations, also known as *object properties*, are dyadic predicates that take individuals as arguments. Similar to how concepts may have a hierarchy based on subsumption, an object property may be a subproperty of another. Object properties can also be inverses of each other, transitive, (ir)reflexive, (a)symmetric. Describing object properties is done also by terminological axioms.

Interpretations are how semantics are provided. An interpretation is a map from individuals to entities in a universe of discourse (also called a *domain*), from concepts to subsets of the universe of discourse, and from object properties to subsets of cartesian products of this universe of discourse.

For the following discussion, let Δ be the universe of discourse, let C, D stand in for some concepts, r, s, t for some object properties, and x, y, z be some individuals mentioned in the *A-box*. Let $\mathcal{I}$ be an interpretation; then the interpretation of C is $C^{\mathcal{I}} \subseteq \Delta$, and analogously we have the interpretations of property r be $r^{\mathcal{I}} \subseteq \Delta \times \Delta$ and of individual x be $x^{\mathcal{I}} \in \Delta$.

Note: DL does not use the unique name assumption. What this means is that the *A-box* may use different names to refer to the same entity in Δ, i.e. $x^{\mathcal{I}} = y^{\mathcal{I}}$.

We will now turn to describing terminological and assertion axioms in more detail. There are several kinds of such axioms, but the ones we require for this paper are:

- is-a assertion, written as $C(x)$, means $x^{\mathcal{I}} \in C^{\mathcal{I}}$
- object property assertion, written as $r(x, y)$, means $(x^{\mathcal{I}}, y^{\mathcal{I}}) \in r^{\mathcal{I}}$
- concept subsumption axiom, written as $C \sqsubseteq D$, means $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
- concept equivalence axiom, written as $C \equiv D$, means $C^{\mathcal{I}} = D^{\mathcal{I}}$
- property subsumption axiom, written as $r \sqsubseteq s$, means $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$
- property inverse axiom, written as $r \equiv s^{-}$, means $\forall \alpha, \beta \in \Delta : (((x^{\mathcal{I}}, y^{\mathcal{I}}) \in r^{\mathcal{I}}) \leftrightarrow ((y^{\mathcal{I}}, x^{\mathcal{I}}) \in s^{\mathcal{I}}))$
- property chain axiom, written as $r \circ s \sqsubseteq t$, means $\forall \alpha, \beta, \gamma \in \Delta : ((((\alpha, \beta) \in r^{\mathcal{I}}) \wedge ((\beta, \gamma) \in s^{\mathcal{I}})) \rightarrow ((\alpha, \gamma) \in t^{\mathcal{I}}))$

Finally, we will briefly present how concept descriptions can be constructed in DL formulas. In the following, let A be a *named*, also known as *atomic* concept. Then, the syntax for a valid concept description can be recursively presented as:

$C := \bot | \top | A | C \sqcap C | C \sqcup C | \exists r.C | \forall r.C$

We will briefly present what each possibility means below:

- the bottom concept, denoted by $\bot$, is the empty concept; for any interpretation, we must have that $\bot^{\mathcal{I}} = \emptyset$
- the top concept, denoted by $\top$, is the concept containing everything; for any interpretation, we must have that $\top^{\mathcal{I}} = \Delta$
- concept intersection, denoted by $C \sqcap D$, is the concept of individuals belonging to both concepts C and D, in other words $(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$

- concept union, denoted by $C \sqcup D$, is the concept of individuals belonging to either concept C or D, in other words $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$
- existential restriction, denoted by $\exists r.C$, is the concept of individuals linking, via property r, to at least one individual of concept C; in other words, $(\exists r.C)^{\mathcal{I}} = \{\alpha \in \Delta | \exists \beta \in C^{\mathcal{I}} : (\alpha, \beta) \in r^{\mathcal{I}}\}$
- universal restriction, denoted by $\forall r.C$, is the concept of individuals that only link, via property r, to individuals of concept C (or to no individuals at all); in other words, $(\forall r.C)^{\mathcal{I}} = \{\alpha \in \Delta | \forall \beta \in \Delta : ((\alpha, \beta) \in r^{\mathcal{I}}) \rightarrow (\beta \in C^{\mathcal{I}})\}$

An observation: it is possible to write the concept description $\exists r.\bot$. This description however also corresponds to the empty concept $\bot$.

Another observation is that an ontology can have many models, depending on what Δ and $\mathcal{I}$ are. E.g., for the same Δ, there may be more ways to map a concept name A to subsets of Δ. However, an ontology can also have no (non-trivial) models at all, if it is possible to infer from it that $\top \sqsubseteq \bot$; such an ontology is said to be inconsistent.

It is useful sometimes to speak of the *domain* or *range* of an object property. If one regards an object property assertion $r(x, y)$ as describing an "arrow" from individual x to individual y, then the domain is the set of individuals that such an arrow may start from, while the range is the set of individuals that it may point to. Using the above syntax and semantics for concept descriptions, the domain and range for property r can respectively be described as:

- $\exists r.\top$
- $\exists r^{-}.\top$

Restricting the domain and range of a property r to, e.g., concepts A and B would then be done by:

- $\exists r.\top \sqsubseteq A$
- $\exists r^{-}.\top \sqsubseteq B$

meaning, "only individuals belonging to concept A may be the start of an r arrow, and only individuals of concept B may be its end". Note, this doesn't mean all individuals of concept A must have an r arrow, nor that all individuals of concept B must receive one. These statements can be expressed as:

- $A \sqsubseteq \exists r.\top$
- $B \sqsubseteq \exists r^{-}.\top$